Low Energy Nuclear Reactions and Sub-Barrier Neutron Transfers

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The problem of anomalous heat production by hydrogenated metals is analysed in the light of deep sub-barrier nucleon transfer reactions. Consideration of the phenomena of condensed matter low energy nuclear reactions and in-vacuum few-body nuclear transfer reactions suggests that LENRs could be due to interactions among the isotopes of hydrogen and certain metals. It is postulated that hydrogen isotope infused heat-producing metals are analogous with in-vacuum ion beam plus metal target systems. It is argued that deep sub-barrier, positive *Q*-value, nucleon transfers among hydrogen and helium isotopes and certain medium and heavy mass metals should occur under condensed matter conditions. It is concluded that several low energy nuclear reaction phenomena cannot yet be excluded as signatures of deep sub-barrier few-nucleon transfers between the nuclei of solvent metals and their dissolved gases. The need for new nuclear models is adumbrated.

Key words: low energy nuclear reaction, neutron transfer, Coulomb barrier, hydrogen isotopes

1 Introduction

Anomalous heat production (AHP) by hydrogenated and deuterated metals is not satisfactorily explained by mainstream nuclear theory. It is now clear that low energy nuclear reactions (LENR) are real but theory has not so far related them and their energy production with other better understood nuclear processes. Theory is yet to unify LENR with other nuclear phenomena. The relevance, for well accepted nuclear phenomena, of the theories that have been devised to explain LENR [1, 2, 3], are minimal or unclear. They have so far failed to connect the two.

The products of LENR observed in the mass regions of hydrogen and electrode metals [4] are consistent with nuclear transformations at both zones of the nuclear landscape. This is consistent with the participation of the nuclei of both mass regions in the nuclear processes and AHP in LENR experiments. Current nuclear theory attributes the initiation of light element nucleosynthesis, of the types reported in association with hydrogenated metals, to the fusion of hydrogen isotopes, which necessarily have to overcome the high energy classical Coulomb barrier. By contrast, the potential for low energy neutron capture by hydrogen isotopes in light element nucleosynthesis is usually omitted from consideration.

This paper proposes: (1) that LENR, observed in the numerous experimental setups that have employed hydrogenated and deuterated metals over the last 16 years [4], involve the nuclei of the metals as well as the gases they contain—each is a participant in the reactions. Nucleon transfer reactions occur between the nuclei of the metals and contained gas nuclei. (2) Gaseous dissolution in the metal behaves as a physical analogue of the in-vacuum systems used in the well-known sub-barrier elastic and quasi-elastic transfer experiments using hydrogen and helium isotope projectiles and medium and high mass nuclear targets. The present theory does not require the fusion of hydrogen isotopes in its explanation of AHP and nuclear transformations in LENR experimental setups.

2 Few-Body Nucleon Transfer Reactions

Neutrons, protons and light nuclei transfer between isotopes of light and heavy nuclei at elastic scattering energies [5]. Heavy and light ion reactions behave differently at deep sub-barrier energies and several heavy ion fusion cross sections fall off steeply at low energies [6]. By contrast, *Q*-values for ground state light ion reactions are often positive. Consequently, few-nucleon transfers can occur down to zero centre-of-mass energy. For a recent treatment of low energy heavy ion fusion data see ref. 6.

Multi-nucleon transfers in the grazing regime, which are frequently associated with the fusion of heavy nuclei, are isotope dependent. The transfers and fusion are seemingly separate components of the global interaction. Deep sub-barrier multi-nucleon transfers are accompanied by considerable energy loss – they are not well understood and cannot be satisfactorily modelled by standard methods or explained by current theory [5, 7]. Barrier penetration calculations are irrelevant to heavy ion fusion cross sections and associated nucleon transfers that occur well below the Coulomb barrier [8].

The theory of nuclear reactions divides the nucleus-nucleus potential into a repulsive Coulomb interaction and a nuclear attractive component. It is clear from the fact that the single barrier penetration model and measured low energy fusion cross sections are orders of magnitude apart [9, 10], that the classical concept of the Coulomb barrier cannot be sustained. The history of our understanding of electric charge is undergoing a significant shift. Electric charge is no longer considered a classical continuous substance, as it was in the 19th century; it is a quantum property of the fermions that are capable of annihilating photons. Again, despite the realization that nucleons behave as quantum systems and not as classical objects, deep sub-barrier transfers are usually explained in a classical framework and resolved by coupling rotational and vibrational energy states to the relative momentum of the colliding systems [11]. An adequate non-classical, theoretical description of these phenomena is not yet to hand [5]. It is clear that the classical concept of a single fixed Coulomb barrier is unable to explain the facts. The alternative view, that barrier penetration is due to quantum effects, is becoming more attractive.

3 In-Vacuum Transfer Data

Nucleon transfer experiments have employed numerous light, medium and heavy nuclei from across the nuclear landscape. Among light elements, isotopes of hydrogen and helium have been shown to donate and pickup neutrons, protons and deuterons in sub-barrier interactions with numerous elements, see Table 1.

Table 1. Examples of nucleon transfers in Au and Pt interactions with isotopes of hydrogen and helium.

Pickup	H and He Transfer Interactions	ref.
1n	^{197}Au(d*,t)^{196}Au	[12]
1n	^{197}Au(p,d)^{196}Au	[12]
d	^{198}Hg(d*,α)^{196}Au	[12, 13]
1n	^{197}Au(^{3}He,^{4}He)^{196}Au	[14]
1n	^{196}Pt(p,d)^{195}Pt	[5, 15]
1n	^{196}Pt(d*,t)^{195}Pt	[5, 15]
Donate		
p	^{194}Pt(^{4}He,t)^{195}Au	[16]
p	^{194}Pt(^{3}He,d)^{195}Au	[16]
1n	^{194}Pt(d*,p)^{195}Pt	[17]

* polarised deuteron, n = neutron, p = proton

Consideration of experimental projectile–target dynamics and the theory of the nucleus-nucleus potential suggest that the element of chief importance is isotope-dependence. The dynamics of those experimental systems suggest that propitious space and phase separations between interacting nuclei are also required for nuclear interactions, including single and multinucleon transfers.

Table 2 shows some examples of neutron transfers in heavy and medium ion interactions. Like sub-barrier heavy ion fusion, the various factors at play in neutron and proton transfer are complex and how they are to be understood is not clear.

Table 2. Examples of neutron transfers in medium and heavy ion scattering and fusion interactions

Pickup	Heavy Ion Interactions	ref.
1-8n	$^{40}Ca + ^{96}Zr$	[18]
nil	$^{40}Ca + ^{90}Zr$	[18]
nil	$^{48}Ca + ^{48}Ca$	[19]
2n,4n	$^{40}Ca + ^{48}Ca$	[19]
2n,4n	$^{62}Ni + ^{206}Pb$	[20]
1n,2n	$^{32}S + ^{197}Au$	[11]
2n	$^{34}S + ^{197}Au$	[11]
nil	$^{36}S + ^{197}Au$	[11]
1n,2n	$^{32}S + ^{208}Pb$	[11]
2n	$^{34}S + ^{208}Pb$	[11]
nil	$^{36}S + ^{208}Pb$	[11]
2n	$^{32}S + ^{110}Pd$	[21]
nil	$^{36}S + ^{110}Pd$	[21]

n = neutron

The Coulomb barrier, which is not characterised by a single, simple energy level, is not an impediment to the transfer of neutrons between nuclei, but it does set limits to the spatial separations of interacting nuclei.

It can be seen from Table 1 that isotopes of hydrogen and helium are able to mediate changes in both proton and neutron number among heavy elements in in-vacuum ion beam experiments. Many of the heavier ion, neutron pickup interactions (Table 2) occur at deep sub-barrier energies. Fundamental principles do not forbid these in-vacuum transfers from taking place in a condensed matter environment. The potentials for the n, p and np transfers in interactions that mediate isotope shifts in the Pt-Hg mass region (shown in Table 1) are positive and occur without an energy input. If propitious geometric relations (PGR) obtain between interacting nuclei the positive Q-value transfers take place and energy is released. Positive Q-values for the nucleon transfers mean that the Coulomb barrier separates the charged atoms and nuclei but does not prevent the transfer of nucleons from donating to receiving nuclei. Positive Q-value transfers presumably depend upon propitious spatial separations and vibration or oscillation phase relations of the reactants. The relative momentum, or beam energy, achieves PGR between projectile and target in the vacuum, but a detailed description at the particle level is yet to be worked out.

One point stands out in these studies—transfers are isotope dependent. An example of that dependence is seen the ^{40}Ca + ^{90,96}Zr systems (Table 2). The chief difference between the two systems is the Q-value for neutron pickup. The ^{90,96}Zr low-lying collective states and deformation parameters are very similar according theory and therefore do not explain the Q-value differences [18]. On the other hand, polarised deuterons may donate a neutron to ^{194}Pt [17], pickup a neutron from ^{197}Au[12] and pickup a deuteron from ^{198}Hg [12, 13]. Final states for these deuteron interactions, which are essentially unexplained, are assumed to have an as yet undetermined structural basis at the level of the metal target nuclei.

As mentioned above, fusion of nuclei at energies below the Coulomb barrier is aided by the isotope-dependent transfer of neutrons between colliding reactants. The transfer is associated with the fusion [22]. An example of this type of synergy is seen with two isotopes of calcium. At deep sub-barrier energies the *Q*-values for all neutron transfer channels of the $^{48}Ca+^{48}Ca$ reaction are negative and the fusion cross section is small. By contrast, the $^{40}Ca+^{48}Ca$ reaction has $Q(2n) = +2.6$ MeV and $Q(4n) = +3.9$ MeV. Zagrebaev proved for the first time that these intermediate neutron transfers are strongly coupled to significantly enhanced sub-barrier fusion cross sections [23]. An example that is relevant to condensed matter LENR is the two-neutron pickup channel for the $^{32}S+^{110}Pd$ reaction which has a large positive ground state *Q*-value (+5.1 MeV) [21, 24].

4 The Relation Between Condensed Matter LENR and In-Vacuum Transfer Reactions

It is postulated that LENR in hydrogenated and deuterated metals has its genesis in nuclear reactions of the same type as discussed above. It follows from that postulate that the condensed matter systems used in AHP experiments during the last 16 years house the same few-nucleon transfer reactions that occur in in-vacuum projectile–target experiments. They are not simply interactions among hydrogen isotopes that occur in isolation from their immediate environments. The reactants are the host metals and dissolved isotopes of hydrogen and/or helium together with other elements whose presence has been shown to influence AHP. Those other elements include lithium and calcium; natural isotopes of both elements readily pickup neutrons. Light nuclear fusion reactions are not part of the present theory.

Fusion of hydrogen and helium isotopes cannot explain the transmutation of heavy elements observed in numerous LENR systems [4]. Hydrogen isotope-dependent transmutation of the metal components of those systems is an explanation of the phenomena which draws on the physics of low energy in-vacuum nuclear transfers. A potential source of ^{4}He is the decay of $^{8,9}Li$ which are products of one- and two-neutron capture by $^{6,7}Li$. By contrast with the low energy sector deuterium fusion theory, the present theory is consistent with the facts of low mass nuclear physics. Helium may also be produced by alpha decay of unstable heavy elements that are transmutation products associated with AHP. Tritium may be produced by neutron capture by either light hydrogen or deuterium.

Intuition suggests that in the above in-vacuum systems, in which positive *Q*-value nucleon transfers occur, the experimental procedure need only achieve PGR among the reactants—they need to be in the right place and have the right phase relations. Ion beams achieve those conditions and there is no fundamental impediment to achieving the same sub-barrier PGR by other means. Techniques that have been successfully employed in AHP include electrolytic ion diffusion, sonocation of gas systems, low pressure gas diffusion and laser photo-stimulation. In addition to achieving PGR among reactants they are each likely to cause shifts in β-decay rates due to altered physical and chemical conditions [25]. Some environment-induced half-life shifts are known but many are yet to be determined.

The hydrogen and helium pickup reactions shown in Table 1 are simply illustrative examples of their generic behaviour. Those particular reactions are not postulated to be associated with significant probabilities at zero centre-of-mass energies. Neither are they necessarily likely to be useful in LENRs. Hydrogen and helium isotopes are postulated to pickup neutrons from some isotopes of metals, including their impurities, employed as electrodes and other components of effective LENR systems. That postulate is consistent with several facts of heat producing LENR systems.

5 Few-Nucleon Transfer Signatures

If LENR were due to transfer reactions among the nuclei of dissolved gases and the solvent metals used for electrodes or components of non-electrolytic condensed matter experimental setups the following phenomena, which are frequently observed in LENR [4], are to be expected:

(1). Light and heavy hydrogen will each react with certain isotopes of various metals to produce heat and transmutated gas and metal isotopes.

(2). Heat generation will be accompanied by nucleosynthesis due to transmutation of a wide range of elements and their isotopes by neutron and light nuclei capture. Post AHP isotope ratios can be expected to deviate from terrestrial values. The dominant novel isotopes are expected to be those related to the host metal by few nucleon transfers and consequent decays.

(3). The lag phase of detected heat production in individual experiments, which is usually attributed to gradual gas accumulation, is consistent with the small proportion of gas atoms that are predicted to randomly achieve PGR with metal nuclei and the consequent slow accumulation of heat-producing, decaying and reacting isotopes.

(4). The duration of excess energy production, in a particular laboratory setup, will be limited by the exhaustion of the metal's potential to react with the dissolved gas by the gradual consumption of its reactive isotopes. In theories that assume that the metal is an inert lattice for LENRs, only the hydrogen isotope is consumed and the decline in AHP with time is not satisfactorily explained. In the present theory, both are consumed but continual replenishment of the gas cannot maintain heat production as the numbers of reactive metal isotopes decline.

(5). In electrolytic cells that have accumulated unstable isotopes, heat production will continue after the current is switched off due to continuing exothermic decay of the unstable isotopes.

(6). The dominant isotope of calcium (^{40}Ca constitutes 97% of the naturally occurring metal) has been shown to be particularly effective in sub-barrier neutron pickup from metal nuclei [18, 19]. It is interesting that the incorporation of CaO in the palladium–deuterium experimental setup of Iwamura et.al. is a necessary condition for nuclear transmutation in that system [26]. According to the present analysis, calcium isotope ratios should also change during the course of those transmutations.

(7). The isotopes of lithium are expected to pickup neutrons and alpha emission is among the consequent isotope decay modes (see above). Lithium is a frequent component of electrolytic LENR systems and ^{4}He has often been detected following AHP in lithium-containing systems. Indeed, it has been reported that ^{4}He detection in electrolytic systems is dependent upon the presence of lithium [27]. Such a dependence is consistent with neutron pickup by ^{6,7}Li nuclei.

According to the present analysis, the above listed features of LENR are the expected consequences of few-nucleon transfer reactions in hydrogen isotope-infused heat producing systems. They relate to the expected signatures of sub-barrier, positive Q-value nucleon transfers between some metals and hydrogen isotopes.

6 The Role of Nuclear Models

Nuclear models serve several functions. They help in the understanding of structure and are the basis for predictions of nuclear behaviour in both novel and disparate conditions. The unification of diverse phenomena within a single theoretical scheme is a prime role for models as well as theory. Currently, different models are employed for the interpretation of different types of nuclear phenomena and are applied in different parts of the nuclear landscape. The ultimate aim of nuclear model building is to embrace the whole of nuclear physics in a single overarching scheme. So far that goal seems rather remote.

Several theoretical models have been developed in an effort to explain apparent deuterium fusion and heavy element transmutation in the low energy sector [1, 2, 3]. Their limitation lies in the absence of the provision of a connection to existing nuclear physics. The present theory is little more than a reinterpretation of the well-known facts of AHP by LENR; it does not propose any new physics. Its chief contribution lies in the connection it proposes between dense matter AHP and mainstream nuclear physics. Condensed matter LENR and in-vacuum transfers are thereby given a possible connection. If the connection is real the factors which constitute necessary conditions for in-vacuum sub-barrier transfers will play a similar role in dense matter reactions. As emphasised above, the physical variable of chief importance for sub-barrier, positive *Q*-value transfers is their isotope dependence. Some isotopes of an element have positive *Q*-values for a particular reaction and others do not and why they differ is not satisfactorily explained by current models. As mentioned above, no model explains the different *Q*-values for the two systems ^{40}Ca + $^{90,96}Zr$ [18], but a structural relationship seems likely.

The interacting boson model of the nucleus (IBM) provides a unified description of nuclear excitations. It is particularly relevant to one- and two-nucleon transfers in the Pt-Au mass region. The dynamical symmetries associated with those interactions have stimulated a reassessment of few-nucleon transfers generally, and the issue of their occurrence in nature is the subject of renewed interest and ongoing investigations [28, 29, 30]. The IBM classifies nuclides as bosons and fermions. Odd-particle nuclei are treated as approximate fermions and even-even nuclei as approximate bosons. Intuition suggests that LENR and the nuclear symmetries uncovered by the IBM will not be unconnected. Those symmetries are possibly the deepest and most fundamental in the whole of nature.

A recent approach to modelling nuclear structure employs two-, three- and four-body clusters in a scheme of discrete, reactive collectivity which extends throughout the whole of the naturally occurring nuclear landscape. It is an extended cluster model (ECM) without a role for shells or other substructures [31]. In that scheme, nuclear degrees of freedom are clusters for all $A>4$ nuclei. Light nuclei in the $A\leq4$ region are individual clusters composed of nucleons. The overall structural arrangement of the set of clusters of each isotope of the naturally occurring elements responds dynamically to neutron addition or removal. Individual two-, three- and four-body clusters, which are the projectiles for the reactions shown in Table 1 are reactive to the addition or removal of neutrons.

In the ECM, one- and two-neutron transfers to and from medium and heavy nuclei are associated with significant nuclear structural change [32]. A double potential for the increase and decrease of nucleon number occurs among the isotopes of several elements that have been shown to transfer nucleons at deep sub-barrier energies. It is not without interest that the clusters which are the central elements of the model and give the M>4 bound nuclei their discrete reactive collectivity, are the same light nuclei of the reactions shown in Table 1.

7 Conclusions

It is argued in this paper that the various dense matter LENR systems, that are known to produce anomalous heat, collectively constitute an analogue of in-vacuum sub-barrier nucleon transfer phenomena, well known in mainstream low-energy nuclear physics.

Recent improved understanding of the conditions which enable in-vacuum sub-barrier transfers, associated with energy loss to the environment, are expected to assist a better understanding and, concomitantly, improved design and performance of heat producing condensed matter LENR systems.

It is clear that traditional models are unable to accommodate many phenomena and data recently derived from both condensed matter and in-vacuum nuclear physics. The possibility that they are two aspects of the same nuclear processes has not been ruled out. New models are required in order to unify the theoretical background for the two experimental approaches and provide the foundation for new and testable predictions. Such models are expected to guide the design of the next generation of LENR systems.

Acknowledgements

I am grateful to Vincent Powell for helpful discussions.

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